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TRANSVERSE NERNST-ETTINGSHAUSEN EFFECT IN A QUANTUM WELL

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We have studied transverse Nernst-Ettingshausen effect in a quantum well (QW) with parabolic potential in the presence of a magnetic field parallel to the plane of the quantum well. The calculation was carried out for the case of elastic electron scattering by acoustic phonons for any degree of degeneration of the electron gas. In the quantum limit the dependencies of the transverse Nernst-Ettingshausen coefficient on the magnetic field strength and the carrier density are determined and analyzed. The weak and strong magnetic fields cases are considered. It is shown that as the consentration increasing the curves of Nernst-Ettingshausen coefficient shift upwards. In the range of weak magnetic fields Nernst-Ettingshausen coefficient increased with the magnetic field.

1. Introduction

Currently, the transport phenomena in systems of reduced dimensionality, such as a quantum well or a quantum wire, is a very practical problem. These systems are of interest due to their potential applications in quantum electronic devices. The size quantization results in the transport phenomena in such systems being sharply different from the transport phenomena in bulk samples. The effect of an in-plane magnetic field on the transport phenomena of an electron gas in a quantum well is even more complicated. In the direction parallel to the magnetic field, the carriers have a free motion, whereas in the plane perpendicular to the magnetic field they are subjected to a combination of magnetic and confinement forces.

The theory of the quantum thermomagnetic effects in size-quantized systems was studied in [1-11]. In [1-2] the conductivity tensor components σ and β are calculated in a quantum wire. The confining potential of a quantum wire was approximated as a parabola. The case of a strongly degenerate electronic gas was considered and a focus was placed on the oscillation phenomena. The thermoelectric power oscillates with the width of the wire and the magnitude of the magnetic field. The important remark to make is that in contrast to the bulk crystal the diagonal components of the conductivity tensor can exceed the non-diagonal ones. The same conclusion was also obtained in [3,11].

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In [4] the thermopower in quantum well structures has been calculated, and the size dependence of thermopower in a quantum limit for different mechanisms of electronic scattering has been considered. In [5] the same authors have calculated the electronic thermopower in quantum wires. In these works the authors used the kinetic equation method and the density matrix approach. In the latter case, the scattering was entered into the equation of motion for the density matrix through the lifetime of a quantum state. In [6] the magneto-thermoelectric power of a two-dimensional electron gas (2DEG) was investigated in the regime of the quantum Hall effect at values of a magnetic field where thermopower is proportional to the entropy of the two-dimensional electron gas. In [7] the magneto-thermoelectric power and Nernst-Ettingshausen effect of a two-dimensional electron gas has been investigated theoretically within the framework of the Boltzman kinetic equation for different mechanisms of electronic scattering taking into account phonon-drag contributions.

Hicks and Dresselhaus [8] predicted that the thermoelectric figure of merit for two-dimensional quantum wells and one-dimensional quantum wires should be substantially enhanced relative to the corresponding bulk materials. A theoretical study of this effect has been undertaken for a bismuth nanowire [15]. In [9] the mechanism for the increase of thermoelectric power of n-type multivalley $PbTe/Pb_{1-x}Eu_xTe$ quantum wells has been investigated theoretically.

The theory of thermopower in quantum dots was developed in [10]. In this work it has been shown that there is an opportunity to create an appreciable temperature difference in a nanostructure and to measure the potential difference induced by this temperature gradient. The paper provides theoretical calculations of magnetothermoelectric power and the Nernst-Ettingshausen effect in quantum wells and quantum wires.

In this paper we have calculated the galvanomagnetic and thermomagnetic tensor components for the current density in a quantum well for any degree of degeneration. It is common knowledge that the thermoelectric tensor has two contributions, diffusion and phonon drag, which are linearly additive. In our work , we are examinated the diffusion component under the assumption that elastic scattering is dominated.

Elastic electron scattering by acoustic phonons is considered. However, the results obtained are also applicable for scattering by short-range impurity potential, roughness of a surface, and alloy-disorder scattering. The magnetic field is directed across the confinement direction, i.e. it is located in the plane of a two-dimensional electron gas. Thus, two cases for the relative arrangement of the current direction and the confinement direction are possible. In the case where the current is located in a plane of a two-dimensional electron gas it is sufficient to confine ourselves to the relaxation time approximation and to use the kinetic equation. In a case when the current is along the direction of confinement it is necessary to use the density matrix approach obtained in [12] and [13] for calculation of the diagonal conduction tensor components.

II. Energy spectrum and wave function

We consider a simple model for the quantum well, in which a two-dimensional electron gas is confined in the x-direction and a homogenous static magnetic field B parallel to the z-axis,with the vector-potential A (0,x B,0) in the Landau gauge.. Then the one-particle Hamiltonian is given by

$$\widehat{H} = \frac{\widehat{p}_x^{\,2}}{2m^*} + \frac{1}{2m^*}\left(\widehat{p}_y + \frac{e}{c}xB\right)^2 + \frac{\widehat{p}_z^{\,2}}{2m^*} + U(x) \tag{1}$$

where $\mathbf{p} = (p_x, p_y, p_z)$ and m^*, respectively, are the momentum operator and the effective mass of a conduction electron. U(x) is the confining potential in the x-direction which is characterized by the parabolic potential:

$$U(x) = \frac{m^* \omega_0^2 x^2}{2} \tag{2}$$

The eigenvalues and eigenfunctions of the Schrodinger equation with Hamiltonian (1) are determined by the expressions

$$\varepsilon_\alpha = \left(\frac{1}{2} + N\right)\omega\hbar + \left(\frac{\omega_0}{\omega}\right)^2 \frac{\hbar^2 k_y^2}{2m^*} + \frac{\hbar^2 k_z^2}{2m^*} \tag{3}$$

$$\phi_\alpha(x, y, z) = \frac{1}{2\pi}\varphi_N(x - x_\alpha)e^{i(k_y y + k_z z)} \tag{4}$$

where $\omega = \sqrt{\omega_0^2 + \omega_c^2}$ - is the "hybrid" frequency, $\omega_c = \frac{eB}{m^* c}$ is the cyclotron frequency of electrons and N - is the oscillation quantum number. The expression

$$\varphi_N(x - x_\alpha) = \frac{1}{\sqrt[4]{\pi}\sqrt{R}\sqrt{2^N N!}} \exp\left(-\left(\frac{x - x_\alpha}{\sqrt{2}R}\right)^2\right) H_N\left(\frac{x - x_\alpha}{R}\right) \tag{5}$$

represents the wave function of a harmonic oscillator, $x_\alpha = -\frac{\omega_c}{\omega}R^2 k_y$ - is the oscillator center, and $R = \sqrt{\frac{\hbar}{m\omega}}$ - is the magnetic length , $H_N(\xi)$ -is the Hermite polynomial, $\alpha = (N, k_y, k_z)$ -is a set of quantum numbers that determine the electron states in a magnetic field.

III. Transverse effects in a quantum well.

In this section we calculate the transverse Nernst-Ettingshausen coefficient Q. The latter consists of the following: If there is a temperature gradient along a conductor which is placed in a magnetic field perpendicular to the temperature gradient, there will appear an electric field perpendicular to the temperature gradient and magnetic field.

For the magnetic field directed along the z-axis the current density components can be written in the form [14]

$$\begin{aligned} j_x &= \sigma_{xx}E_x + \sigma_{xy}E_y - \beta_{xx}\nabla_x T - \beta_{xy}\nabla_y T \\ j_y &= \sigma_{yx}E_x + \sigma_{yy}E_y - \beta_{yx}\nabla_x T - \beta_{yy}\nabla_y T \end{aligned} \tag{6}$$

where σ_{ik} and β_{ik} are the conduction tensor components, E_k is the components of the electric field and $\nabla_k T$ is the temperature gradient.

From the conditions $j_x = j_y = 0$, $\nabla_x T = 0$ we obtain the transversal Nernst-Ettingshausen effect

$$Q_{xy} = -\frac{E_x}{B\nabla_y T} = -\frac{1}{B}\frac{\beta_{xy}\sigma_{yy} - \beta_{yy}\sigma_{xy}}{\sigma_{xx}\sigma_{yy} - \sigma_{xy}\sigma_{yx}} \tag{7}$$

Putting $j_x = j_y = 0$, $\nabla_y T = 0$ we obtain from Eqs. (6)

$$Q_{yx} = \frac{E_y}{B\nabla_x T} = \frac{1}{B}\frac{\beta_{yx}\sigma_{xx} - \beta_{xx}\sigma_{yx}}{\sigma_{xx}\sigma_{yy} - \sigma_{xy}\sigma_{yx}} \tag{8}$$

For the calculation of kinetic coefficients Q_{xy} , Q_{yx} it is necessary to calculate both diagonal and non-diagonal conduction tensor components σ_{ik} and β_{ik} .

Note that in bulk semiconductors $\sigma_{yx} >> \sigma_{xx}$, $\beta_{yx} >> \beta_{xx}$, $\sigma_{xy} >> \sigma_{yy}$ and $\beta_{xy} >> \beta_{yy}$ [13]. It is related to the fact that a decrease in scattering potential results in the diagonal electric conductivity tensor components tending to zero, while the non-diagonal components stay finite.

A. Calculation of the non-diagonal components of the thermomagnetic and galvanomagnetic tensors.

The average value of the current density components carried by the electrons is defined by the expression

$$j_i = -eTr\left(\hat{\rho}\hat{v}^{(i)}\right) = -e\sum_{\alpha\alpha'}\rho_{\alpha\alpha'}v^{(i)}_{\alpha'\alpha} \quad , \quad i = (x, y, z) \tag{9}$$

where $\hat{\rho}$ is the density matrix, and $\hat{\mathbf{v}}$ - is the velocity operator.

The matrix elements of the density matrix are evaluated from the solution of Liouville's equation

$$i\hbar\frac{\partial\hat{\rho}}{\partial t} = \left[H_t, \rho\right] \tag{10}$$

where $\hat{H}_t$ is the total Hamiltonian of the system $\hat{H}_t = \hat{H} + V + F$ which consists of the Hamiltonian (1), the scattering potential V, and the electron-electric field interaction $F = e(\mathbf{E}\cdot\mathbf{r})$.

The matrix elements of the components of the velocity operator in the representation (4) can be written as

$$\hat{v}^x_{\alpha\alpha'} = i\omega R\,\delta_{k_y,k_y'}\delta_{k_z,k_z'}\left(\delta_{N',N-1}\sqrt{\frac{N}{2}} - \delta_{N',N+1}\sqrt{\frac{N+1}{2}}\right) \tag{11}$$

$$\hat{v}^y_{\alpha\alpha'} = \omega_c R\,\delta_{k_y,k_y'}\delta_{k_z,k_z'}\left(\delta_{N',N+1}\sqrt{\frac{N+1}{2}} - \delta_{N',N-1}\sqrt{\frac{N}{2}}\right) + \frac{\omega_0^2\,\hbar k_y}{m^*\omega^2}\delta_{\alpha,\alpha'} \tag{12}$$

$$\hat{v}^z_{\alpha\alpha'} = \frac{1}{m^*}\hbar k_z\delta_{\alpha,\alpha'} \tag{13}$$

Using Eqns. (11)-(13) in Eq.(9) and performing the summation over $\alpha' = \left(N', k_y', k_z'\right)$ we obtain the following expressions for the current density components:

$$j_x = -ie\omega R \sum_{\alpha} \left(\rho_{N-1,N} \sqrt{\frac{N}{2}} - \rho_{N+1,N} \sqrt{\frac{N}{2}} \right) \tag{14}$$

$$j_y = -e\,\omega_c R \sum_{\alpha} \left(\rho_{N+1,N} \sqrt{\frac{N+1}{2}} + \rho_{N-1,N} \sqrt{\frac{N}{2}} \right) - \frac{e\hbar}{m^*} \left(\frac{\omega_0}{\omega} \right)^2 \sum_{\alpha} k_y \, \rho_{\alpha,\alpha} \tag{15}$$

$$j_z = -e \frac{\hbar}{m} \sum_{\alpha} k_z \, \rho_{\alpha,\alpha} \tag{16}$$

In a zero-order approximation with respect to the scattering potential V the matrix elements of the density matrix $\rho_{\alpha,\alpha'}$ have the form

$$\rho_{\alpha',\alpha} = e\left(E_x x_{\alpha'\alpha} + E_y y_{\alpha'\alpha}\right) \frac{f_{\alpha'} - f_{\alpha}}{\varepsilon_{\alpha'} - \varepsilon_{\alpha}} \tag{17}$$

where $x_{\alpha'\alpha}$ and $y_{\alpha'\alpha}$ are the matrix elements of the x and y coordinates, respectively. In Eqn.(17) $f_{\alpha} = f\left(\varepsilon_{\alpha}\right)$ is the equilibrium electron distribution function (Fermi-Dirac function)

$$f\left(\varepsilon_{\alpha}\right) = \left(1 + \mathbf{exp}\left(\frac{\varepsilon_{\alpha} - \zeta}{k_0 T} \right) \right)^{-1} \tag{18}$$

where ζ is the chemical potential of the electrons, and k_0 is the Boltsmann constant.

Substituting Eq.(17) into Eqs.(14)-(15) and calculating the matrix elements of the coordinates we obtain

$$j_y = \sigma_{yx} E_y, \; \sigma_{yx} = \frac{\omega_c e^2}{m^* \omega^2} \sum_{\alpha} f_{\alpha} = \frac{\omega_c e^2 n}{m^* \omega^2}, \tag{19}$$

$$j_x = 0$$

where $\sum_{\alpha} f_{\alpha} = n$ is the areal density of the two-dimensional electron gas and

$$n = \frac{k_0 T m^*}{2\pi\hbar^2} \frac{\omega}{\omega_0} \sum_{N} \ln\left(1 + e^{\eta - x_N}\right), \tag{20}$$

where

$$\eta = \frac{\zeta}{k_0 T}, \qquad x_N = \frac{\hbar\omega}{k_0 T}\left(N + \frac{1}{2} \right). \tag{21}$$

In the limit of strong magnetic fields, $\omega_0 << \omega_c$, or equivalently, in the bulk case,

when $\omega_0 \to 0$ the energy spectrum (3) equals that of an electron in a magnetic field. In this case the expression for σ_{yx} in (19) coincides with that for the non-diagonal component of the conductivity tensor of the bulk semiconductor material [14].

In order to find the explicit form of the non-diagonal component $\beta_{xy}(B)$ we will take advantage of the Onsager reciprocal relation

$$\beta_{xy}(B) = \frac{1}{T}\gamma_{yx}(-B) \tag{22}$$

where $\gamma_{ik}(B)$ is the coefficient in the formula of i-th heat flux desity transported by the electrons $W_i = \gamma_{ik} E_k$.

In Ref.[16] it was shown explicitly that in the presence of a magnetic field it is necessary to take into account the contribution to the current of electrons the edge current $-c\,\nabla \times \mathbf{M}$ due to magnetization $\mathbf{M}$. In this case the coefficient γ_{yx} can be represented as

$$\gamma_{yx} = \gamma_{xy}^{(0)} - cM \tag{23}$$

where $\gamma_{yx}^{(0)}$ is the coefficient in the heat flux density in the absence of scattering which defined by [17; 1]

$$W_y^0 = \frac{1}{2}\sum_{\alpha\alpha'}\rho_{\alpha'\alpha}v_{\alpha\alpha'}^{y}\left(\varepsilon_\alpha + \varepsilon_{\alpha'} - 2\zeta\right) \tag{24}$$

The magnetization M is defined by the relationship $M = -\left(\frac{\partial\Omega}{\partial B}\right)_{T,\zeta}$, where $\Omega = -k_0 T\sum_\alpha \mathbf{ln}\left(1 + \mathbf{exp}\left(\frac{\zeta - \varepsilon_\alpha}{k_0 T}\right)\right)$ is the Gibbs thermodynamic potential.

Substitution of the Eq.(17) into Eq.(24) yields the following expression for $\gamma_{yx}^{(0)}$

$$\gamma_{yx}^{(0)} = -\frac{e\omega_c R^2}{\hbar\omega}\left(\bar{\varepsilon} - \zeta n + \sum_\alpha \hbar\omega\left(N + \frac{1}{2}\right)f_\alpha\right) \tag{25}$$

where $\bar{\varepsilon} = \sum_\alpha \varepsilon_\alpha f_\alpha$ is the average energy of the system.

The magnetization can be written as

$$M = -\frac{\omega_c^2}{\omega^2}\frac{\Omega}{B} - \frac{1}{B}\frac{\omega_c^2}{\omega^2}\sum_\alpha \hbar\omega\left(N + \frac{1}{2}\right)f_\alpha \tag{26}$$

where

$$\Omega = -\left(k_0 T\right)^2 \frac{\omega}{\omega_0}\frac{m}{2\pi\hbar^2}\sum_N \int_{x_N}^{\infty} \mathbf{ln}(1 + \mathbf{exp}(\eta - x))dx \tag{27}$$

is the thermodynamic potential per unit area.

Using (26) we obtain the following expression for $\gamma_{yx}^{(0)}$ instead of Eqn.(25):

$$\gamma_{yx}^{(0)} = -\frac{e\omega_c R^2}{\hbar\omega}\left(\bar{\varepsilon} - \zeta n - \frac{\omega^2}{\omega_c^2}MB - \Omega\right) \tag{28}$$

On the other hand, according to the definition of the thermodynamic potential

$$\Omega = \overline{\varepsilon} - \zeta n - TS \tag{29}$$

where $S = -\left(\frac{\partial \Omega}{\partial T}\right)_{B,\zeta}$ is the entropy per unit area which has the following form:

$$S = \frac{mk_0^2 T}{2\pi\hbar^2}\frac{\omega}{\omega_0}\sum_N \left((x_N - \eta)\ln\left(1 + e^{\eta - x_N}\right) - 2Li_2\left(-e^{\eta - x_N}\right)\right) \tag{30}$$

where $Li_\nu(\xi)$ is the polylogarithmic function of order ν .

Substituting (29) into (28) and using (23) we obtain

$$\gamma_{yx} = -\frac{e\omega_c}{m\omega^2}TS \tag{31}$$

Finally, for β_{xy} we obtain

$$\beta_{xy} = \frac{e\omega_c}{m\omega^2}S \tag{32}$$

Similar expression was obtained in Ref.[1] for a quantum wire.

For the strong magnetic field case, $\omega_c \gg \omega_0$, Eqn. (32) is reduced to $\beta_{xy}^{(bulk)} = cS / B$, which was obtained in Ref.[16] for bulk specimens. At zero temperature, the transport coefficient β_{xy}, consequently, the entropy vanishes as required by the third law of thermodynamics.

B. Transverse diagonal components of the thermomagnetic and galvanomagnetic tensors

For the calculation of the diagonal components of tensors α_{xx} and β_{xx} when the electric field or the temperature gradient are perpendicular to the plane of two-dimensional electron gas we will make use of the expressions obtained in [12] and [13]:

$$\beta_{xx} = -\frac{e}{\Omega_0 T}\sum_{\alpha\alpha'}\left(-\frac{\partial f(\varepsilon_\alpha)}{\partial \varepsilon_\alpha}\right)\frac{(x_{\alpha'} - x_\alpha)^2}{2}(\varepsilon_\alpha - \zeta)W_{\alpha\alpha'} \tag{33}$$

$$\sigma_{xx} = \frac{e^2}{\Omega_0 T}\sum_{\alpha\alpha'}\left(-\frac{\partial f(\varepsilon_\alpha)}{\partial \varepsilon_\alpha}\right)\frac{(x_{\alpha'} - x_\alpha)^2}{2}W_{\alpha\alpha'} \tag{34}$$

where Ω_0 is the volume of the system, $W_{\alpha\alpha'}$ is the electron transition probability from state $\alpha = \left(N, k_y, k_z\right)$ to state $\alpha' = \left(N', k_y', k_z'\right)$ caused by the effect of the scattering potential.

The scattering mechanism explicitly considered in the present paper is the acoustic phonon deformation potential (DPA scattering). Acoustic-phonon scattering via piezoelectric coupling could also be considered, but this will have a similar temperature and magnetic-field dependence to DPA scattering and so is not included separately. because will not qualitatively affect the results. Other scattering

mechanisms, as the interface roughness mechanism, playes a negligible role in heterojunctions, because with the current crystal growth metods, high crystalline quality with atomically sharp resolution is easily achieved, that is interface are not especially rough. In addition, impurity scattering arising from background impurities in the quantum well or remote ionized donors it was to be expected, however in high magnetic fields the magnetic length will be much smaller than the scale of fluctuations due the remote impurities, so remote impurity scattering can be treated by the short-range point defect approach. In this case, the scattering from the point defects has the same functional form as for scattering from the DPA. Only, the temperature and electron concentration dependence will be different.

The transition probability due to the carrier scattering by acoustic phonons has the form

$$W_{\alpha\alpha'} = \sum_{\vec{q}} w(q) \left| \left\langle \alpha \left| e^{iq_x x + iq_y y + iq_z z} \right| \alpha' \right\rangle \right|^2 \times \\ \times \left(N_q \delta_{k_y', k_y + q_y} \delta_{k_z', k_z + q_z} \delta\left(\varepsilon_{\alpha'} - \varepsilon_\alpha - \hbar\omega_q\right) + \left(N_q + 1\right) \delta_{k_y', k_y - q_y} \delta_{k_z', k_z - q_z} \delta\left(\varepsilon_{\alpha'} - \varepsilon_\alpha + \hbar\omega_q\right) \right) \tag{35}$$

where

$$w(q) = \frac{\pi E_1^2}{\rho s \Omega_0} q \tag{36}$$

Here s is the speed of sound, ρ is the density of the material, E_1 is the constant of the acoustic phonon deformation potential, q is the phonon wave vector, and $N_q = \left(\mathbf{exp}\left(\frac{\hbar\omega_q}{k_0 T} \right) - 1 \right)^{-1}$ is the occupation number (the Planck function) for phonons with frequency $\omega_q = s\,q$.

Using the wavefunctions from Eqn.(4), one can write the matrix elements of the electron - phonon interaction as

$$\left\langle \alpha \left| e^{i\mathbf{q}\cdot\mathbf{r}} \right| \alpha' \right\rangle \Big|^2 = \left| J_{NN'}\left(q_x, q_y\right) \right|^2 \delta_{k_y', k_y + q_y} \delta_{k_z', k_z + q_z} \tag{37}$$

where

$$\left| J_{NN'}\left(q_x, q_y\right) \right|^2 = \frac{N!}{N'!} \mathbf{exp}\left(-\frac{R^2\left(q_x^2 + q_y^2 \frac{\omega_c^2}{\omega^2}\right)}{2} \right) \left(\frac{R^2\left(q_x^2 + q_y^2 \frac{\omega_c^2}{\omega^2}\right)}{2} \right)^{N'-N} \left(L_N^{N'-N}\left(\frac{R^2\left(q_x^2 + q_y^2 \frac{\omega_c^2}{\omega^2}\right)}{2} \right) \right)^2 \tag{38}$$

Here $L_n^m(\xi)$ is the associated Laguerre polynomial.

Further we will focus on the extreme situation, namely, the quantum limit in which the scattering of electrons is confined within the $N = N' = 0$ level. For the quantum well in a magnetic field the quantum limit criterion is $\hbar\omega > k_0 T$.

Above 20 K the available acoustic phonon energies will be small compared to $k_0 T$. Since the electron scattering by the acoustic phonons is elastic, it is possible to neglect the phonon energy $\hbar\omega_q$ in the arguments of the δ -functions in (35). In addition, as $\hbar\omega_q < k_0 T$, therefore it is possible to expand the Plank function. Thus

we obtain

$$2N_q + 1 \approx \frac{2k_0T}{\hbar sq} \tag{39}$$

Taking Eqn. (39) into account we can rewrite the expression for σ_{xx} in the form

$$\sigma_{xx} = \frac{e^2}{\Omega_0}\frac{w_0}{2}\left(\frac{\omega_c}{\omega}R^2\right)^2 \sum_{q_x}\sum_{k_z,k_y}\sum_{k_y',k_z'}\left(-\frac{\partial f}{\partial \varepsilon}\right)\left(k_y' - k_y\right)^2 e^{-\frac{R^2\left(q_x^2+\left(\frac{\omega_c}{\omega}\right)^2\left(k_y'-k_y\right)^2\right)}{2}} \times$$
$$\times\delta\left(\frac{\hbar^2 k_z'^2}{2m^*} + \left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar^2 k_y'^2}{2m^*} - \frac{\hbar^2 k_z^2}{2m^*} - \left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar^2 k_y^2}{2m^*}\right), \tag{40}$$

here

$$w_0 = \frac{4\pi T E_1^2 k_0}{s^2\rho\Omega_0\hbar} \tag{41}$$

Transforming the sum over q, k_y, k_y', k_z and k_z' in Eq.(40) into an integral form in a usual way we introduce new, deformed coordinates,

$$k_y' = \frac{\omega}{\omega_0}\tilde{k}_y',\ k_y = \frac{\omega}{\omega_0}\tilde{k}_y,\ k_z' = \tilde{k}_z',\ k_z = \tilde{k}_z \tag{42}$$

Using the momentum conservation law $\tilde{k}' = \tilde{k}$ and integrating over the angle between vectors $\vec{\tilde{k}}'$ and $\vec{\tilde{k}}$ we obtain

$$\sigma_{xx} = \frac{1}{\tau_0}\frac{3e^2 n}{m^*\omega_0^2}\left(\frac{\omega_c}{\omega_0}\right)^2\sqrt{\frac{\omega_0}{\omega}}\frac{1}{\ln\left(1+e^{\tilde{\eta}}\right)}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)x\ {}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8ax\right)dx. \tag{43}$$

Similarly, for β_{xx} we obtain

$$\beta_{xx} = -\frac{k_0}{e}\frac{1}{\tau_0}\frac{3e^2 n}{m^*\omega_0^2}\left(\frac{\omega_c}{\omega_0}\right)^2\sqrt{\frac{\omega_0}{\omega}}\frac{1}{\ln\left(1+e^{\tilde{\eta}}\right)}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)x\left(x-\tilde{\eta}\right)\ {}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8ax\right)dx \tag{44}$$

In Eqns.(43) and (44) the following notations were used

$$\tau_0 = \frac{\sqrt{2\pi}\ s^2\rho\hbar^{7/2}}{m^{*3/2}\ E_1^2 k_0 T\sqrt{\omega_0}} \tag{45}$$

$$a = \frac{k_0 T\omega_c^2}{2\omega\hbar\omega_0^2} \tag{46}$$

$$f_0 = \left(1+\exp\left[x-\tilde{\eta}\right]\right)^{-1},\quad \tilde{\eta} = \eta - x_0 \tag{47}$$

and ${}_2F_2\left(a_1,a_2;b_1,b_2;z\right)$ is the generalized hypergeometric function [18].

C. Diagonal components ot the thermomagnetic and galvanomagnetic tensors along a plane of two-dimensional electron gas

For the case of the electric field and the temperature gradient directed along the plane of two-dimensional electron gas, we use the solution of the kinetic equation to calculate the diagonal components of the tensors α_{yy} and β_{yy} . These are given by

$$\sigma_{yy}=\frac{e^2}{\Omega_0}\sum_{\alpha}\left(-\frac{\partial f\left(\varepsilon_\alpha\right)}{\partial\varepsilon_\alpha}\right)\tau_\alpha v_{k_y}^2 \tag{48}$$

$$\beta_{yy}=-\frac{e^2}{\Omega_0 T}\sum_{\alpha}\left(-\frac{\partial f\left(\varepsilon_\alpha\right)}{\partial\varepsilon_\alpha}\right)\left(\varepsilon_\alpha-\xi\right)\tau_\alpha v_{k_y}^2 \tag{49}$$

where

$$v_{k_y}=\frac{1}{\hbar}\partial_{k_y}\varepsilon=\left(\frac{\omega_0}{\omega}\right)^2\frac{\hbar k_y}{m^*} \tag{50}$$

and τ_α is the relaxation time of electrons.

For the elastic scattering by acoustic phonons the relaxation time is given by a simple expression:

$$\frac{1}{\tau_\alpha}=\sum_{\alpha'}W_{\alpha\alpha'}\left(1-\frac{\mathrm{k}'}{\mathrm{k}}\right) \tag{51}$$

Taking into account expression (35) for the transition probability and proceeding the same was as above in the calculation of the transverse diagonal components of the transport coefficients, the following expression for τ obtains in the quantum limit:

$$\tau=\tau_0\left(\frac{\omega_0}{\omega}\right)^{3/2}\frac{1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-\frac{4\,\omega_e^2}{\omega\hbar\omega_0^2}\left(\varepsilon-\frac{\hbar\omega}{2}\right)\right)} \tag{52}$$

Substitution of (52) into (48) and (49) and summation over α yields

$$\sigma_{yy}=\frac{e^2 n}{m^*\ln\left(1+e^{\tilde{\eta}}\right)}\tau_0\left(\frac{\omega_0}{\omega}\right)^{7/2}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)\frac{1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8ax\right)}xdx \tag{53}$$

$$\beta_{yy}=-\frac{k_0}{e}\frac{e^2 n}{m^*\ln\left(1+e^{\tilde{\eta}}\right)}\tau_0\left(\frac{\omega_0}{\omega}\right)^{7/2}\int_0^\infty\left(-\frac{\partial f_0}{\partial x}\right)\frac{\left(x-\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8ax\right)}xdx \tag{54}$$

Combining Eqns.(19), (32), (43), (44), (53) and (54) with Eqns. (7)-(8) and taking into account the symmetry of the conductivity tensor, we obtain the following expressions:

$$Q_{xy} = -\frac{\tau_0 \omega^2 \left(\frac{\omega_0}{\omega}\right)^{7/2} \ln\left(1+e^{\tilde{\eta}}\right)}{Be n \omega_c} \frac{\left(S K_3 - k_0 n K_4\right)}{3K_1 K_3 + \ln^2\left(1+e^{\tilde{\eta}}\right)}, \tag{55}$$

$$Q_{yx} = -\frac{3\omega^{3/2} \omega_c \ln\left(1+e^{\tilde{\eta}}\right)}{B e n \tau_0 \omega_0^{7/2}} \frac{\left(S K_1 - k_0 n K_2\right)}{3K_1 K_3 + \ln^2\left(1+e^{\tilde{\eta}}\right)}, \tag{56}$$

where

$$K_1 = \int_0^\infty \left(-\frac{\partial f_0}{\partial x}\right) {}_2F_2\left(\frac{5}{4}, \frac{7}{4}; \frac{3}{2}, 2; -8ax\right) x\, dx \tag{57}$$

$$K_2 = \int_0^\infty \left(-\frac{\partial f_0}{\partial x}\right) x\left(x-\tilde{\eta}\right) {}_2F_2\left(\frac{5}{4}, \frac{7}{4}; \frac{3}{2}, 2; -8ax\right) dx \tag{58}$$

$$K_3 = \int_0^\infty \left(-\frac{\partial f_0}{\partial x}\right) \frac{1}{{}_2F_2\left(\frac{1}{4}, \frac{3}{4}; \frac{1}{2}, 1; -8ax\right)} x\, dx \tag{59}$$

$$K_4 = \int_0^\infty \left(-\frac{\partial f_0}{\partial x}\right) \frac{\left(x-\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4}, \frac{3}{4}; \frac{1}{2}, 1; -8ax\right)} x\, dx \tag{60}$$

Eqns.(55)-(56) is applicable for any degree of degeneracy of two-dimensional electron gas.

IV. Transverse Nernst-Ettingshausen effect for a strongly degenerated electron gas

For the degenerated electron gas, when $\eta = \zeta/k_0T > 1$, the expressions for the transport coefficients can be considerably simplified. In this case we replace $\left(-\frac{\partial f(\varepsilon)}{\partial \varepsilon}\right)$ by the delta function $\delta(\in -\zeta)$. Then we obtain

$$n = \frac{m^* k_0 T}{2\pi\hbar^2} \frac{\omega}{\omega_0} \tilde{\eta}, \tag{61}$$

$$S^{(d)} = \frac{\pi m^* T k_0^2 \omega}{6\hbar^2 \omega_0}, \tag{62}$$

$$\sigma_{yx}^{(d)} = \frac{\omega_c e^2 n}{m^* \omega^2}, \tag{63}$$

$$\beta_{yx}^{(d)} = -\frac{k_0}{e} \frac{\pi^2}{3\tilde{\eta}} \sigma_{yx}^{(d)} \tag{64}$$

$$\sigma_{xx}^{(d)} = \frac{1}{\tau_0} \frac{3e^2 n}{m\omega_0^2} \left(\frac{\omega_c}{\omega_0}\right)^2 \sqrt{\frac{\omega_0}{\omega}}\ {}_2F_2\left(\frac{5}{4}, \frac{7}{4}; \frac{3}{2}, 2; -8a\tilde{\eta}\right) \tag{65}$$

$$\beta_{xx}^{(d)} = -\frac{k_0}{e}\frac{\pi^2}{3\tilde{\eta}}\sigma_{xx}^{(d)}\left(1-\frac{35}{6}\frac{a\tilde{\eta}\ {}_2F_2\left(\frac{9}{4},\frac{11}{4};\frac{5}{2},3;-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2,-8a\tilde{\eta}\right)}\right) \tag{66}$$

$$\sigma_{yy}^{(d)} = \frac{e^2 n}{m}\tau_0\left(\frac{\omega_0}{\omega}\right)^{7/2}\frac{1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8a\tilde{\eta}\right)} \tag{67}$$

$$\beta_{yy}^{(d)} = -\frac{k_0}{e}\frac{\pi^2}{3\tilde{\eta}}\sigma_{yy}^{(d)}\left(1+\frac{3a\tilde{\eta}\ {}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2,-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8a\tilde{\eta}\right)}\right) \tag{68}$$

For the case of strongly degenerate electron gas the Nernst-Ettingshausen coefficients can be written as

$$Q_{xy} = \frac{\pi^2\omega^2 k_0\tau_0\left(\frac{\omega_0}{\omega}\right)^{7/2}}{3e\omega_c B}\frac{3a\Psi_1}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8a\tilde{\eta}\right)\left(3\Psi_1+1\right)} \tag{69}$$

$$Q_{yx} = -\frac{35\pi^2\,\omega^{3/2}k_0\omega_c}{6B\,e\,\tau_0\,\omega_0^{7/2}}\frac{\Psi_2\ a\ {}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2,-8a\tilde{\eta}\right)}{\left(3\Psi_1+1\right)} \tag{70}$$

where

$$\Psi_1 = \frac{{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2,-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{1}{4},\frac{3}{4};\frac{1}{2},1;-8a\tilde{\eta}\right)} \tag{71}$$

$$\Psi_2 = \frac{{}_2F_2\left(\frac{9}{4},\frac{11}{4};\frac{5}{2},3;-8a\tilde{\eta}\right)}{{}_2F_2\left(\frac{5}{4},\frac{7}{4};\frac{3}{2},2;-8a\tilde{\eta}\right)} \tag{72}$$

In the case of strongly degenerate electron gas and weak fields, $\tilde{\eta}$ depends on the magnetic field only weakly, and the observed changes in the transport coefficients are due to parameter a . Indeed, from the expansion of Q_{xy} with respect to a ,

$$Q_{xy} = \frac{k_0^2\,\pi^2\,T\,\tau_0\omega_0\sqrt{\frac{\omega_0}{\omega}}}{8c\,\omega^2\hbar\,m^*}\left(1+\frac{55n\pi\hbar\omega_c^2}{24\,\omega^2\omega_0 m^*}\right) \tag{73}$$

one can see that in a weak field this coefficient can grow at sufficiently high concentrations and fall at sufficiently low concentrations. Furthermore, from (70)

$$Q_{yx} = -\frac{35\left(k_0\pi^2\omega^2\sqrt{\frac{\omega_0}{\omega}}\omega_c\right)a}{24\left(Be\tau_0\omega_0^4\right)} \tag{74}$$

In fact, Q_{yx} tends to zero in a weak field, in contrast to Q_{xy} which remains constant. In a strong magnetic field the situation is similar to the quantum limit for the bulk case.

V. Results and discussions

In this section we present our numerical calculations for the transverse Nernst-Ettingshausen coefficient for $GaAs/Al_xGa_{1-x}As$ parabolic quantum well. We use the following set of physical parameters $m^* = 0.066 m_0$, where m_0 is the free electron mass. The parameter of the parabolic potential is $\omega_0 = 1.4 \times 10^{13}\ s^{-1}$. The value of the deformation-potential constant is as $E_1 = 10\ eV$. The density of the material and the speed of sound are taken as $\rho = 5 \times 10^3\ kg/m^3$ and $s = 5400\ m/s$. Notice that numerical calculations for the quantum limit criterion must be carried out when the Fermi level is between the first and second subbands $\hbar\omega/2 \le \zeta < 3\hbar\omega/2$.

The dependence of the transverse Nernst - Ettingshausen effect Q_{xy} upon the magnetic field at T=20 K for various values of the two - dimensional concentration is shown in Fig.1. The temperature gradient is along the direction of the free motion. One can see that Q_{xy} is monotonically decreasing with the magnetic field,in the range 1-10 Tesla,. As the concentration increasing the curves Q_{xy} shift upwards. In the range of weak magnetic fields (0.01–0.03 Tesla) Qxy increased with the magnetic field (did not shown on Fig.1).

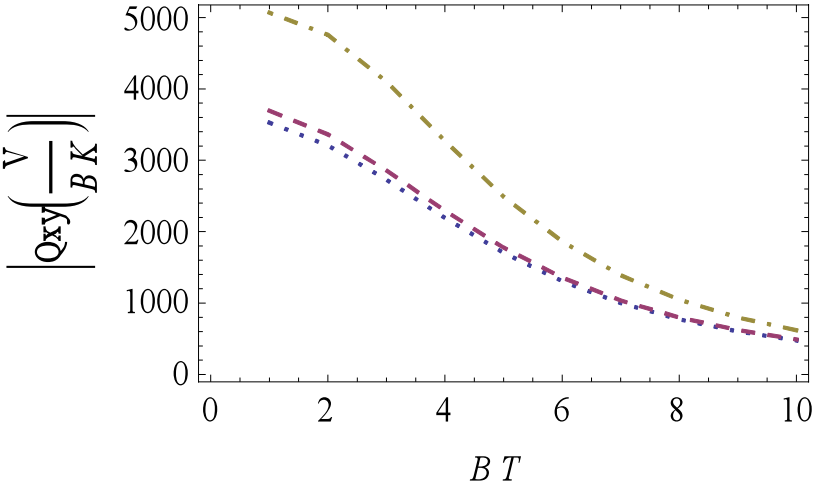

Fig.1. Absolute value transversal Nernst-Ettingshausen coefficient Q_{xy} of two-dimensional electron gas versus the magnetic field for various values of the concentrations at T=20K. $n=10^{13}$ m^{-2} – dotted, $n=10^{14}$ m^{-2} – dashed , $n=10^{15}$ m^{-2} – dotdashed.

The dependence of the transverse Nernst - Ettingshausen effect Q_{yx} upon the magnetic field at T=20 K for various values of the two - dimensional concentration, when temperature gradient is along the direction of confinement is shown in Fig .2. We see that Q_{yx} increases with the magnetic field As the concentration increasing the curves Q_{xy} shift upwards when magnetic field lower that 4 Tesla, and shift down for

more strong magnetic field.

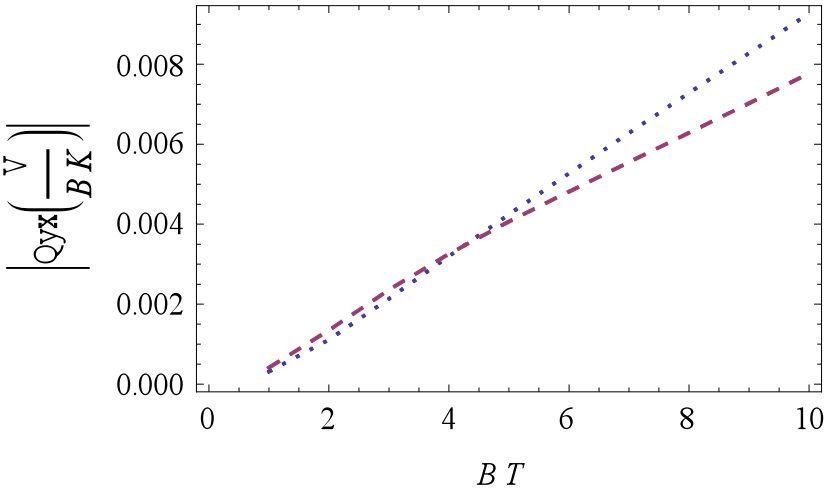

Fig.2. Absolute value transversal Nernst-Ettingshausen coefficient Q_{yx} of two-dimensional electron gas versus the magnetic field for two values of the concentrations at T=20K. $n=10^{14}$ m^{-2} – dotted, $n=10^{15}$ m^{-2} – dashed.

As temperature increases Q_{yx} monotonically decrease, whereas the temperature dependence of Q_{xy} is non - monotonic at low concentrations. At concentrations above $n=10^{11}cm^{-2}$ the pattern of the temperature dependence changes, and in the strong degenerate case the coefficients do not depend on the temperature (see formulae for the strongly degenerate case).

VI. Conclusion

The transverse Nernst - Ettingshausen effect in a quantum well (QW) with parabolic potential in the presence of a magnetic field parallel to the plane of the quantum well have been calculated. The calculation has been carried out for the case of elastic electron scattering by acoustic phonons for any degree of degeneration of the electron gas. In the quantum limit the dependencies of the transverse Nernst - Ettingshausen coefficient on the magnetic field strength, the temperature and the carrier density have been determined and analyzed.

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KVANT ÇUXURUNDA ENİNƏ NERNST-ETTİNQSHAUZEN EFFEKTİ

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XÜLASƏ

Bu işdə parabolik potensiala malik kvant çuxurunda, onun müstəvisinə paralel istiqamətdə maqnit sahəsi olduqda eninə Nernst-Ettinqshauzen effekti öyrənilmişdir. Hesablamalar elektronların akustik fononlardan elastik səpilməsi və istənilən dərəcədə cırlaşmanın olduğu hal üçün aparılmışdır. Kvant limitində eninə Nernst-Ettinqshauzen əmsalının maqnit sahəsinin intensivliyindən və yük sıxlığından asılılığı müəyyən olunmuş və analiz edilmiş, zəif və güclü maqnit sahələri hallarına baxılmışdır. Göstərilmişdir ki, konsentrasiyanın artması ilə Nernst-Ettinqshauzen əmsalının əyriləri yuxarıya doğru sürüşür.

ПОПЕРЕЧНЫЙ ЭФФЕКТ НЕРНСТА-ЭТТИНГСГАУЗЕНА В КВАНТОВОЙ ЯМЕ

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РЕЗЮМЕ

Мы изучили поперечный эффект Нернста-Эттингсгаузена в квантовой яме с параболическим потенциалом в присутствии магнитного поля, расположенного в плоскости квантовой ямы. Вычисления были выполнены для случая упругого рассеяния электронов акустическими фононами при любой степени вырождения электронного газа. Определена и анализирована зависимость поперечного коэффициента Нернста-Эттингсгаузена от напряженности магнитного поля и концентрации носителей в квантовом пределе. Рассмотрены случаи слабых и сильных магнитных полей. Показано что при увеличении концентрации кривые зависимости коэффициента Нернста-Эттингсгаузена от поля смещаются вверх. В случае слабого магнитного поля при увеличении магнитного поля коэффициент Нернста-Эттингсгаузена увеличивается.